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The KN Algebras and the Geometrical Quantization of Heterotic Strings

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By means of the KN algebra, the Virasoro and Kac–Moody superalgebras are extended on a genus-g Riemann surface. These are realized in the framework of heterotic superstring with the WZ term coupling the fiber bundle. Then we construct the corresponding BRST operators, which provide the critical dimension.

1. INTRODUCTION

Since Krichever and Novikov introduced KN algebra (Krichever *et al.*, 1987*a,b*), work extending this algebra into conformal fields has been very successful (Bonora *et al.*, 1988*a,b*; Xu and Zhao, 1990). Krichever and Novikov introduce a new formalism which is a strong tool. These works consider the algebra (to be referred to as KN algebra) in a general formalism of meromorphic vector fields which are holomorphic outside two distinguished points and introduce an explicit countable basis. In terms of these basis elements, one can develop an operator out of an arbitrary Riemann surface while preserving an explicit dependence on the genus. Moreover, some authors have studied bosonic strings and superstrings on a genus-g Riemann surface and recovered the critical dimensions for any genus remain 26 and 10 for the bosonic string and superstring, respectively (Liu *et al.*, 1990; Harari, 1987). Liu *et al.* (1990) discussed the case when the motion space of the string is $M^d \times G$, where M^d is a d-dimensional

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Minkowski space and G is a d_G-dimensional group manifold. By constructing a BRST charge using the method of geometrical quantization, Liu *et al.* (1990) studied the Sugawara construction and gave the critical dimension.

The organization of this paper is as follows: In Section 2 the KN algebra and the scheme of geometrical quantization are reviewed and the heterotic superstring WZ term coupling the fiber bundle is introduced. In Section 3 the curvatures of the holomorphic bundle of the string and a vacuum bundle are calculated. In conclusion, in Section 4 we perform the quantization procedure for a heterotic superstring coupling a fiber bundle.

2. THE KN ALGEBRA AND GEOMETRICAL QUANTIZATION

We consider a Riemann surface of genus g with two distinguished points P_+, P_- and local coordinates Z_+, Z_- around them, such that $Z_\pm(P_\pm) = 0$. There exists a whole family of tensors $f_j^{(\lambda,x)}$ parametrized by two real numbers λ (the conformal weight) and x. The $f_j^{(\lambda,x)}$ are holomorphic everywhere on Σ except possibly for poles or branch points in P_+ and P_- and a branch cut from P_+ to P_-. This can be considered as an extended case of a Kahler manifold existing usually on a Riemann surface Σ with genus zero. Moreover, the expansion of $f_j^{(\lambda,x)}$ near $P_\pm$ is of the form

$$f_j^{(\lambda,x)} = a_j^{(\lambda,x)\pm} Z_{\pm}^{\pm j + x}{}_{-S(\lambda)} \pm [1 + O(Z_\pm)](dz_\pm)^\lambda \tag{2.1}$$

where $S(\lambda) = g - \lambda(g-1)$ and $a_i^{(\lambda,x)\pm}$ are constants to be specified below. In particular, for different values of λ and x, we obtain the meromorphic vector fields of the function $A_j = f_j^{(0,0)}$, the one-differentials $\Omega_j = f_j^{(2,0)}$, respectively, which are holomorphic except at $P_\pm$ (the definitions of A_j and w_j have to be slightly modified for $|j| \leqq \frac{1}{2}g$ due to the Weierstrass theorem). According to whether g is even or odd, j is an integer, $j = \cdots, -1, 0, 1, \ldots,$ or half-integer, $j = \cdots, -\frac{1}{2}, \frac{1}{2}, \ldots,$ respectively. On the other hand, we introduce the meromorphic spinor fields $g_\alpha = g_j^{(-1/2,0)}$ when $\alpha =$ integer and $f_{(j+1/2)}^{(-1/2,1/2,0)}$ when $\alpha = j + 1/2$, the half-differentials $h_{-\alpha} = f_{-j}^{(1/2,0)}$ when $\alpha = j$ integer and $h_{-\alpha} = f_{(j+1/2)}^{(1/2,1/2)}$ when $\alpha = j + 1/2$ half-integer, and the 3/2-differentials $K_\alpha = f_{(j+1/2)}^{(3/2,1/2)}$ when $\alpha = j + 1/2$ half-integer, where α is an integer (Ramond sector) or half-integer (Neveu–Schwarz sector). Due to the Riemann–Roch theorem, the above basic elements are determined up to an arbitrary constant. So, we normalize them by setting $a_i^{(\lambda,0)} = 1$; $a_i^{(\lambda,x)}$ will then be uniquely determined as well as all the coefficients appearing in the tails $O(Z_\pm)$.

Let us come now to the bilinear operations which will allow us to define a superalgebra. It follows from an analysis of the singularities in $P_\pm$

that (Bonora *et al.*, 1988*a*)

$$[e_t, e_j] = \sum_{z=-g_0}^{g_0} C^z_{tj} e_{t+j-z} g_0 = \frac{3}{2} g \tag{2.2a}$$

$$[e_t, g_\alpha] = \sum_{z=-g_0}^{g_0} H^z_{t\alpha} g_{t+\alpha-z}. \tag{2.2b}$$

$$\{g_\alpha, g_\beta\} = \sum_{p=-g}^{g} B^\alpha_{\alpha\beta} e_{\alpha+\beta-p/2} \tag{2.2c}$$

The coefficients C^z_{tj}, $H^z_{t\alpha}$, and $B^p_{\alpha\beta}$ can be calculated from the constants appearing in the expansion of e_i and g_α near $P_\pm$; for example, in the simplest case, we have $C^{g_0}_{ij} = j - i$, $H^{g_0}_{i\alpha} = \alpha - i/2 - g + g_0/2$, $B^g_{\alpha\beta} = 2$. It is easy to check that the following duality relations hold:

$$\frac{1}{2\pi i} \oint_{C_\tau} A_t(Q) W_j(Q) = \delta_{tj} \tag{2.3a}$$

$$\frac{1}{2\pi i} \oint_{C_\tau} e_t(Q) \Omega_j(Q) = \delta_{tj} \tag{2.3b}$$

$$\frac{1}{2\pi i} \oint_{C_\tau} g_\alpha(Q) K_\beta(Q) = g_{\alpha\beta} \tag{2.3c}$$

$$\frac{1}{2\pi i} \oint_{C_\tau} h_\alpha(Q) b^+_\beta(Q) = \delta_{\alpha\beta} \tag{2.3d}$$

where $h^+_\alpha = h_{-\alpha}$ and C_τ denoted a level curve of the univalent function $\tau(Q)$ (Liu *et al.*, 1990). The central extensions of these algebras are defined by means of cocycles

$$\chi(e_t, e_j) = \frac{1}{24\pi i} \oint \tilde{\chi}(e_t, e_j) \tag{2.4a}$$

$$\varphi(g_\alpha, g_\beta) = \frac{1}{6\pi i} \oint \tilde{\varphi}(g_\alpha, g_\beta) \tag{2.4b}$$

where the integral is over a contour surrounding P_+ and $\tilde{\chi}$, $\tilde{\varphi}$ are defined as follows. Let f, g and ρ, σ be meromorphic vector fields and meromorphic spinor fields, respectively, which are holomorphic on Σ except possibly for poles or branch points in $P_\pm$ (with associated branch cuts), and let

$$f = f(Z_\pm)\,\partial/\partial Z_+, \qquad g = g(Z_+)\,\partial/\partial Z_+$$

$$\rho = \rho(Z_+)(dZ_+)^{-1/2}, \qquad \sigma = \sigma(Z_+)(dZ_+)^{-1/2}$$

near P_+; then one finds

$$\tilde{\chi}(f, g) = \left[\frac{1}{2}(f'''g - g'''f) - R(f'g - g'f)\right] dZ_+ \tag{2.5a}$$

$$\tilde{\phi}(\rho, \sigma) = \rho'\sigma'\, dZ_+ \tag{2.5b}$$

where R is a Schwarzian connection (Liu *et al.*, 1990).

Thus the central extension of both NS-KN and R-KN superalgebras is as follows:

$$[e_i, e_j] = \sum_{z=-g_0}^{g_0} C_{ij}^z e_{i+j-zg_0} + \chi(e_i, e_j) \tag{2.6a}$$

$$[e_i, g_\alpha] = \sum_{z=-g_0}^{g_0} H_{i+\alpha-z}^z \tag{2.6b}$$

$$\{g_\alpha, g_\beta\} = \sum_{p=-g}^{g} B_{\alpha\beta}^p e_{\alpha+\beta-p/2} + t\varphi(g_\alpha, g_\beta) \tag{2.6c}$$

$$[e_i, t] = [g_\alpha, t] = 0 \tag{2.6d}$$

which reduce to the usual Neveu–Schwarz and Ramond superalgebras in the genus-zero case.

We can calculate the cocycles χ and φ in a few cases. For example, for $k = 0$,

$$\chi(e_i, e_{3g-i}) = \frac{1}{12}(i - g_0)^3 - (i - g_0) \tag{2.7a}$$

$$\varphi(g_\alpha, g_{2g-\alpha}) = \frac{1}{3}(\alpha - g)^2 + \frac{1}{12} \tag{2.7b}$$

3. THE CURVATURE OF THE HOLOMORPHIC BUNDLE OF THE STRING AND A VACUUM BUNDLE

Next, we consider the energy-momentum tensor of a heterotic superstring with the WZ term coupling the fiber bundle (Xu *et al.*, 1990; Bergshoeff *et al.*, 1986*a*,*b*)

$$T = -\partial X^\mu\, \partial X^\mu - \frac{1}{2}\partial X^\mu \chi^\mu - \frac{1}{2}\partial\zeta^\mu\zeta^\mu - \frac{1}{2}J^a J^a - \frac{1}{2}\partial\varphi^a\varphi^a - \frac{1}{2}\partial\psi^I\psi^I \tag{3.1}$$

and the supersymmetric current

$$F = \sqrt{2}X^\mu\, \partial X^\mu + \frac{i}{6}\sqrt{k} f^{abc}\varphi^a\varphi^b\varphi^c \tag{3.2}$$

Here the manifold of the heterotic superstring is taken as a product of d-dimensional Minkowski space M_d with the d_G-dimensional G, $M_d \times G$; J^a is the conserved current on G and χ is its supersymmetric partner. The constant is defined by $\delta_{ab} C_A = f_{acd} f^{cd}_{b}$ with f^{abc} being the structure constants of group G. Let X^μ, χ^μ, ζ^μ, J^a, φ^a, ψ^I, T, F have conformal weights 0, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, 1, $\frac{1}{2}$, $\frac{1}{2}$, 2 and $\frac{3}{2}$, respectively. For the geometrical quantization, suppose φ is a multiplet of spin $-1/2$ fields holomorphic outside $P^\pm$, transforming according to a real representation of G, and let T^a denote the antisymmetric matrices representing the generators of G. Thus, it follows a quantum level that

$$J^a = \frac{1}{2} :\psi(Q) T^a \psi(Q): = \frac{1}{2} :\psi'(Q)(T^a)\psi'(Q): \tag{3.3}$$

where T^a satisfies

$$[T^a, T^b] = \frac{1}{2} f^{abc} T^c \tag{3.4a}$$

$$\mathrm{Tr}(T^a T^b) = -K_1 \delta^{ab} \tag{3.4b}$$

$$C_2 \delta^{cd} = f^{abc} F^{abd} \tag{3.4c}$$

where f^{abc} is the structure constant of group G.

We are now in a position to expand the tensor fields in (3.1) in terms of the relevant bases as follows:

$\lambda = 0$:

$$X^\mu(Q) = \sum_t X^\mu_t A_t(Q) \tag{3.5a}$$

$$X^\mu_t = \frac{1}{2\pi i} \oint_{C_\tau} X^\mu(Q) \omega(Q) \tag{3.5b}$$

$\lambda = \frac{1}{2}$:

$$\chi^\mu(Q) = \sum_\alpha d^\mu_\alpha h_\alpha(Q) \tag{3.5c}$$

$$d^\mu_\alpha = \frac{1}{2\pi i} \oint_{C_\tau} \chi^\mu(Q) h^+_\alpha(Q) \tag{3.5d}$$

$$\zeta^\mu(Q) = \sum_\alpha t^\mu_\alpha h_\alpha(Q) \tag{3.5e}$$

$$t^\mu_\alpha = \frac{1}{2\pi i} \oint_{C_\tau} \zeta^\mu(Q) h^+_\alpha(Q) \tag{3.5f}$$

$$\varphi^{\alpha}(Q) = \sum_{\alpha} s_{\alpha}^{\mu} h_{\alpha}(Q) \tag{3.5g}$$

$$s_{\alpha}^{\mu} = \frac{1}{2\pi i} \oint_{C_\tau} \varphi^{\alpha}(Q) h_{\alpha}^{+}(Q) \tag{3.5h}$$

$$\psi^{I}(Q) = \sum_{\alpha} u_{\alpha}^{I} h_{\alpha}(Q) \tag{3.5i}$$

$$u_{\alpha}^{I} = \frac{1}{2\pi i} \oint_{C_\tau} \psi^{I}(Q) h_{\alpha}^{+}(Q) \tag{3.5j}$$

$\lambda = 1$:

$$dX^{\mu}(Q) + P^{\mu}(Q) = \sqrt{2} \sum_{n} \alpha_{n}^{\mu} \omega_{n}(Q) \tag{3.5k}$$

$$\sqrt{2}\alpha_{n}^{\mu} = \frac{1}{2\pi i} \oint_{C_\tau} [dX^{\mu}(Q) + p^{\mu}(Q)] A_{n}(Q) \tag{3.5l}$$

$$J^{a}(Q) = \sum_{n} J_{n}^{a} \omega_{n}(Q) \tag{3.5m}$$

$$J_{n}^{a} = \frac{1}{2\pi i} \oint_{C_\tau} J^{a}(Q) A_{n}(Q) \tag{3.5n}$$

$$P^{\mu}(Q) = \sum_{n} P_{n}^{\mu} \omega_{n}(Q) \tag{3.5o}$$

$$P_{n}^{\mu} = \frac{1}{2\pi i} \oint_{C_\tau} P^{\mu}(Q) A_{n}(Q) \tag{3.5p}$$

$\lambda = \frac{3}{2}$:

$$F(Q) = \sum_{\alpha} F_{\alpha} K_{\alpha}(Q) \tag{3.5q}$$

$$F_{\alpha} = \frac{1}{2\pi i} \oint_{C_\tau} F(Q) g_{\alpha}(Q) \tag{3.5r}$$

$\lambda = 2$:

$$T(Q) = \sum_{n} L_{n} \Omega_{n}(Q) \tag{3.5s}$$

$$L_{n} = \frac{1}{2\pi i} \oint_{C_\tau} T(Q) e_{n}(Q) \tag{3.5t}$$

where P^{μ} is the conjugate momentum of X^{μ}. The Poisson brackets of these tensors read

$$[X^{\mu}(Q), P^{\nu}(Q')] = i^{-1}\eta^{\mu\nu}\Delta_{\tau}(Q, Q') \tag{3.6a}$$

$$\{\chi^{\mu}(Q), \chi^{\nu}(Q')\} = i^{-1}\eta^{\mu\nu}\delta_{\tau}(Q, Q') \tag{3.6b}$$

$$\{\zeta^{\mu}(Q), \zeta^{\nu}(Q')\} = i^{-1}\eta^{\mu\nu}\delta_{\tau}(Q, Q') \tag{3.6c}$$

$$\{\varphi^{a}(Q), \varphi^{b}(Q')\} = i^{-1}\eta^{ab}\delta_{\tau}(Q, Q') \tag{3.6d}$$

$$\{\varphi^{I}(Q), \psi^{J}(Q')\} = i^{-1}\eta^{IJ}\delta_{\tau}(Q, Q') \tag{3.6e}$$

$$[J^{a}(Q), J^{b}(Q')] = \frac{1}{\sqrt{k}} f^{abc} J^{c}(Q)\Delta_{\tau}(Q, Q') \tag{3.6f}$$

where

$$\Delta_{\tau}(Q, Q') = \sum_{n} A_{n}(Q)\omega_{n}(Q') \tag{3.7}$$

$$\delta_{\tau}(Q, Q') = \sum_{h} h_{n}(Q)h_{n}^{+}(Q') \tag{3.8}$$

play the role of δ-function over C_{τ} for smooth tensors with weight 0 and 1/2, respectively.

Owing to (2.1), one can obtain the Poisson brackets for the coefficients of the mode expansion:

$$[X_{n}^{\mu}, P_{m}^{\nu}] = i^{-1}\eta^{\eta\nu}\delta_{nm} \tag{3.9a}$$

$$\{\alpha_{n}^{\mu}, \alpha_{m}^{\nu}\} = i^{-1}\eta^{\mu\nu}r_{nm} \tag{3.9b}$$

$$\{d_{\alpha}^{\mu}, d_{\beta}^{\nu}\} = i^{-1}\eta^{\mu\nu}\delta_{\alpha+\beta,0} \tag{3.9c}$$

$$\{t_{\alpha}^{\mu}, t_{\beta}^{\nu}\} = i^{-1}\eta^{\mu\nu}\delta_{\alpha+\beta,0} \tag{3.9d}$$

$$\{s_{\alpha}^{a}, s_{\beta}^{b}\} = i^{-1}\eta^{ab}\delta_{\alpha+\beta,0} \tag{3.9e}$$

$$\{u_{\alpha}^{I}, u_{\beta}^{J}\} = i^{-1}\eta^{IJ}\delta_{\alpha+\beta,0} \tag{3.9f}$$

$$[J_{m}^{a}, J_{n}^{b}] = \frac{1}{\sqrt{k}} f^{abc}\alpha_{mn}^{z}J_{z}^{c} + i^{-1}\delta^{ab}r_{mn} \tag{3.9g}$$

where

$$r_{mn} = \frac{1}{2\pi i}\oint_{C_{\tau}} A_{m}\, dA_{n} \tag{3.10a}$$

$$\alpha_{mn}^{z} = \frac{1}{2\pi i}\oint_{C_{\tau}} A_{m}A_{n}\omega_{z} \tag{3.10b}$$

Thus, we have

$$
\begin{aligned}
L_m &= \frac{1}{2\pi i}\oint_{C_\tau} e_n(Q)T(Q) \\
&= -\frac{1}{2}\sum_{nl} l^m_{nl}\alpha^\mu_n\alpha^\mu_l + \frac{1}{4}\sum_{\alpha\beta} D^m_{\alpha\beta}d^\mu_\alpha d^\mu_\beta + \frac{1}{4}\sum_{\alpha\beta} D^m_{\alpha\beta}t^\mu_\alpha t^\mu_\beta \\
&\quad -\frac{1}{2}\sum_{nl} A^m_{nl}J^a_n J^a_l + \frac{1}{4}\sum_{\alpha\beta} D^m_{\alpha\beta}s^a_\alpha s^a_\beta + \frac{1}{4}\sum_{\alpha\beta} D^m_{\alpha\beta}u^I_\alpha u^I_\beta
\end{aligned} \tag{3.11}
$$

$$
F_\alpha = \sum_{n\beta} E^\alpha_{\alpha\beta}\alpha^\mu_n d^\mu_\beta + \frac{1}{4}\sum_{n\beta} E^\alpha_{n\beta}J^a_n s^a_\beta - \frac{i}{6\sqrt{k}}\sum_{\beta\gamma\delta} f^{abc}s^a_\beta s^b_\gamma s^c_\delta G^\alpha_{\beta\gamma\delta} \tag{3.12}
$$

where

$$
A^m_{nl} = \frac{1}{2\pi i}\oint_{C_\tau} \omega_n\omega_l e_m \tag{3.13a}
$$

$$
D^m_{\alpha\beta} = \frac{1}{2\pi i}\oint_{C_\tau} (h_\beta\partial h_\alpha - h_\alpha\partial h_\beta)e_m \tag{3.13b}
$$

$$
E^\alpha_{n\beta} = \frac{1}{2\pi i}\oint_{C_\tau} h_\beta\omega_n g_\alpha \tag{3.13c}
$$

$$
G^\alpha_{\beta\gamma\delta} = \frac{1}{2\pi i}\oint_{C_\tau} h_\beta h_\gamma h_\delta g_\alpha \tag{3.13d}
$$

Here α, β, γ, and δ are either integers (Ramond sector) or half-integers (Neveu–Schwarz sector). Therefore, we may form the Poisson brackets of L_m and F_α as follows:

$$
[L_m, L_n] = i^{-1}\sum_{z=-g_0}^{g_0} C^z_{mn}L_{m+n+-z} \tag{3.14a}
$$

$$
[L_m, F_\alpha] = i^{-1}\sum_{z=-g_0}^{g_0} H^z_{m\alpha}F_{m+\alpha+-z} \tag{3.14b}
$$

$$
\{F_n, F_\beta\} = i^{-1}\sum_{p=-g_0}^{g_0} B^p_{\alpha\beta}L_{\alpha+\beta-p/2} \tag{3.14c}
$$

These relations reduce to the usual one in the genus-zero case. In the quantum case we must take care of operator ordering; L_n will be modified as

$$\hat{L}_m = -\frac{1}{2}\sum_{nl} l_{nl}^m :\alpha_n^\mu \alpha_l^\mu: + \frac{1}{4}\sum_{\alpha\beta} D_{\alpha\beta}^m :d_\alpha^\mu d_\beta^\mu: + \frac{1}{4}\sum_{\alpha\beta} D_{\alpha\beta}^m :t_\alpha^\mu t_\beta^\mu:$$
$$- \frac{1}{2(C_v + K)}\sum_{nl} A_{nl}^m :J_n^a J_l^a: + \frac{1}{4}\sum_{\alpha\beta} D_{\alpha\beta}^m :s_\alpha^a s_\beta^\alpha: + \frac{1}{4}\sum_{\alpha\beta} D_{\alpha\beta}^m :u_\alpha^I u_\beta^I:$$
$$= -\frac{1}{2}\sum_{nl} l_{nl}^m :\alpha_n^\mu \alpha_l^\mu: + \frac{1}{4}\sum_{\alpha\beta} D_{\alpha\beta}^m :d_\alpha^\mu d_\beta^\mu: + \frac{1}{4}\sum_{\alpha\beta} D_{\alpha\beta}^m :t_\alpha^\mu t_\beta^\mu:$$
$$+ \frac{1}{4}\sum_{\alpha\beta} D_{\alpha\beta}^m :s_\alpha^a s_\beta^a: + \frac{1}{4}\sum_{\alpha\beta} D_{\alpha\beta}^m :u_\alpha^I u_\beta^I:$$
$$- \frac{1}{8(C_v + k)}\sum A_{pq}^m E_{\alpha\beta}^p E_{\gamma\delta}^q (T^a)_{tj}(T^a)_{kl} :\psi_\alpha^t \psi_\beta^j \psi_\gamma^k \psi_\delta^l: \tag{3.15}$$

where

$$E_{\alpha\beta}^n = \frac{1}{2\pi i}\oint_{C_\tau} h_\alpha h_\beta A_n(Q)$$

From now on, we state briefly the geometrical quantization (Liu *et al.*, 1990). Take a complex Kahler manifold Γ and define the Poisson brackets as

$$[Z_A, Z_B] = i^{-1}\delta_{AB} \tag{3.16}$$

The two-form in a Kahler manifold Γ is defined as

$$\omega = i\sum_A dZ_A \wedge d\tilde{Z}_A = i\partial\tilde{\partial}K \tag{3.17}$$

where

$$K = \sum_A Z_A \tilde{Z}_A$$

is the Kahler potential. Next, we define a Hilbert space H spanned by ψ endowed an inner product as

$$\langle\psi_1, \psi_2\rangle = \oint \tilde{\psi}_1 \psi_2 \det\omega \tag{3.18}$$

Then the third step amounts to a Kahler polarization procedure, i.e., to putting a constraint on ψ:

$$\psi = e^{-K/2}\Phi, \qquad \partial\Phi = 0 \tag{3.19}$$

The above condition (3.17) means that Φ is a holomorphic function on Γ. The function ψ spans the prequantum Hilbert space H_p and contains the quantum wave function.

4. THE GEOMETRICAL QUANTIZATION

We use the BRST scheme to quantize this model. First, a BRST charge operator on Σ is defined corresponding to the NS-KN and R-KN superalgebras:

$$Q = \sum_n :L_n C_n: + \sum_\alpha F_{-\alpha}\gamma_\alpha: - \frac{1}{2}\sum_{m,n}\sum_{z=-\delta_0}^{g-0} C^z_{mn} :C_{-m}C_{-n}b^{m+n-z}:$$
$$-\sum_{n,\alpha}\sum_{z=-g_0}^{\delta_0} H^z_{m\alpha} :\gamma_{-\alpha}C_{-n}\beta_{\alpha+\beta-\bar{z}}: - \sum_{\alpha,\beta}\sum_{p=-z_0}^{\delta_0} B^z_{m\alpha} :\gamma_{-\alpha}\gamma_{-\beta}b_{\alpha+\beta-p/2}:$$
$$-ac \tag{4.1}$$

Here, the constant a takes into account the ambiguity in normal ordering of the operators, and b_n, C_n and β_α, γ_α are the conformal and superconformal ghosts on a Riemann surface with genus g, respectively. The following anticommutation and commutation relations hold:

$$\{C_n, C_m\} = \delta_{m+n,0} \tag{4.2}$$

$$[\gamma_\alpha, \beta_\beta] = \delta_{\alpha+\beta,0} \tag{4.3}$$

After the quantization, the algebra does not close, one has

$$L'_m = -\frac{1}{2}\sum_{n,l} A^m_{nl} :\alpha^\mu_n \alpha^\mu_l: + \frac{1}{4}\sum_{\alpha,\beta} D^m_{\alpha\beta} :d^\mu_\alpha d^\mu_\beta: - \frac{1}{2}\frac{1}{(1+C_A/2k)}\sum_{nl} A^m_{nl} :J^\alpha_n J^\alpha_l:$$
$$+\frac{1}{4}\sum_{\alpha\beta} D^m_{\alpha\beta} :C^\alpha_n s^\alpha_\beta: \tag{4.4a}$$

$$F'_\alpha = \sum_{n,\beta} E^\alpha_{\mu\beta} :\alpha^\mu_n d^\mu_\beta: + \frac{1}{(1+C_A/2k)^{1/2}}\left(\sum_{n\beta} E^\alpha_{n\beta} :J^a_n s^d_\beta:\right.$$
$$\left. - \frac{1}{6\sqrt{k}}\sum_{\beta\gamma\delta} G^\alpha_{\beta\gamma\delta} f^{abc} s^a_\beta :s^b_\gamma s^c_\delta\right) \tag{4.4b}$$

Then, in virtue of the charge operator Q, we may define new operators $\hat{L}$ and $\hat{F}$:

$$\hat{L}_n = \{Q, b_n\} = L'_n + \sum_m \sum_{z=g_0}^{g_0} C^z_{mn} C_{-m} b_{m+n-z}$$
$$-\sum_\alpha \sum_{z=-g_0}^{g_0} H^z_{n\alpha}\gamma_{-\alpha}\beta_{\alpha+n-z} - a\delta_{n,0} \tag{4.5a}$$

$$\hat{F}_\alpha = [Q, \beta_\alpha] = F'_\alpha - \sum_z \sum_{z=-g_0}^{g_0} H^z_{n\alpha} :C_{-n}\beta_{\alpha+n-z}$$
$$-\sum_\alpha \sum_{z=-g_0}^{g_0} B^p_{\alpha\beta} :\gamma_{-\beta} b_{\alpha+\beta-p/2}: \tag{4.5b}$$

After a tedious (but simple) calculation, we arrive at

$$[\hat{L}_m, \hat{L}_n] = \sum_{z=-g_0}^{g_0} C^z_{mn} \hat{L}_{m+n-z} + \hat{\chi}_{mn} \tag{4.6a}$$

$$[\hat{L}_m, \hat{F}_\alpha] = \sum_{z=-g_0}^{g_0} H^z_{m\alpha} \hat{F}_{m+\alpha-z} \tag{4.6b}$$

$$\{\hat{F}_\alpha, \hat{F}_\beta\} = \sum_{z=-g_0}^{g_0} B^p_{\alpha\beta} \hat{L}_{\alpha+\beta-p/2} + \hat{\varphi}_{\alpha\beta} \tag{4.6c}$$

where the anomalous terms can be expressed as

$$\hat{\chi} = \left\{d - 5 - \frac{1}{3} d_G - \frac{2}{3} \frac{d_G}{1 + C_A/2k}\right\} \chi(e_m, e_n) \tag{4.7a}$$

$$\hat{\varphi}_{\alpha\beta} = \left\{d - \frac{1}{3} - \frac{2}{3} \frac{d_G}{1 + C_A/2k}\right\} \varphi(g_\alpha, g_\beta) \tag{4.7b}$$

When

$$a = \begin{cases} 1 & \text{for NS sector} \quad (4.8a) \\ 1 - \frac{1}{5}(d-2) - \frac{1}{5} d_G & \text{for R sector} \quad (4.8b) \end{cases}$$

$$I = 10 + \frac{1}{3} d_G + \frac{2}{3} d_G \frac{1}{1 + C_A/2k} \tag{4.8c}$$

the nilpotential condition for the BRST charge Q, $Q^2 = 0$, implies the critical dimension of the motion space of a heterotic superstring with the WZ term coupling the fiber bundle. If a and I satisfy (4.8a) or (4.8b) and (4.8c), respectively, we get the critical dimension

$$d = 5 - \frac{1}{3} d_G - \frac{2}{3} \frac{d_G}{1 + C_A/2k} \tag{4.9}$$

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